

10.3: Parametric Tests

Pearson Product-Moment Correlation Test

Purpose

The Pearson product-moment correlation test (Pearson r) is a parametric procedure that determines the strength and direction of the linear relationship between two continuous variables. Pearson r is symmetric, with the same coefficient value obtained regardless of which variable is the IV and which is the DV. It has a value in the range $-1 \leq r \leq 1$. The absolute value of Pearson r can be interpreted as follows (Hinkle, Wiersma, & Jurs, 1998):

- Little if any relationship $< .30$
- Low relationship = $.30$ to $< .50$
- Moderate relationship = $.50$ to $< .70$
- High relationship = $.70$ to $< .90$
- Very high relationship = $.90$ and above

A scatterplot can be used to visually inspect the degree and direction of linear relationship between the two variables. Since Pearson r is symmetric, it makes no difference which variables are displayed on the x- and y-axes. Additionally, one can determine if there is a curvilinear (i.e., nonlinear) component to the relationship that is not captured by the Pearson r correlation coefficient. For example, the relationship between human height and age is mostly a curvilinear relationship over a person's lifespan that would not be adequately captured by Pearson r (points on the scatterplot tend to fall along an arc rather than a straight line) since people increase in height until approximately age 20 and then stop growing. However, the relationship would be mostly linear if the target population were children. The eta correlation coefficient can be used to determine the total degree of relationship (linear plus curvilinear) between two variables.

SPSS data entry for this test is fairly straightforward. Each variable is entered in SPSS Spreadsheet View as a separate column.

Pearson r is calculated by taking cross-products of z-scores.

$$r = \frac{\sum Z_x Z_y}{N}$$

The z-scores in the formula standardize the units of measure in both distributions.

Degrees of freedom. The degrees of freedom for this test is $N - 2$, where N is the number of cases in the analysis.

Alternative Tests

Point-biserial correlation coefficient – parametric; determines symmetric linear relationship between one dichotomous variable and one continuous variable.

Spearman rank order correlation coefficient – nonparametric; determines monotonic symmetric relationship between two ranked variables.

Eta correlation coefficient – nonparametric; determines total linear and nonlinear asymmetric relationship between one nominal and one interval/ratio variable.

Kendall's tau-b – nonparametric; determines monotonic symmetric relationship between two ordinal variables, used when the number of rows and number of columns are equal, adjusts for tied pairs, based on concordant-discordant pairs.

Kendall's tau-c – nonparametric; determines monotonic symmetric relationship between two ordinal variables, used when the number of rows and number of columns are not equal, based on concordant-discordant pairs.

Key Assumptions & Requirements

Random selection of samples (probability samples) to allow for generalization of results to a target population.

Variables. Two interval/ratio scale variables. Many researchers support the use of this test with ordinal scale variables that have several levels of responses. For example, Nunnally and Bernstein (1994) assert that this test can be used with ordinal level variables that have more than 11 rank values.

Absence of restricted range (i.e., data range is not truncated in any variable). Range restriction occurs when one or both variables are restricted in their values. For example, suppose one wants to know the strength of relationship between Graduate Record Examination (GRE) scores and graduate grade point average (GPA). The result will likely be an artificially low correlation coefficient due to restriction of range in GRE – only higher scoring students are accepted for graduate enrollment – and restriction of range in GPA – graduate students with low GPA will drop out of the graduate program so there will be a disproportionate number of low GPA students (if any) in the sample.

Measurement without error.

Bivariate normality. Both variables should have an underlying distribution that is bivariate normal. It indicates that scores on one variable are normally distributed for each value of the other variable, and vice versa. Univariate normality of both variables does not guarantee bivariate normality. A circular or symmetric elliptical pattern in a bivariate scatterplot is evidence of a bivariate normal distribution. The professional literature is mixed regarding the robustness of Pearson r to non-normality, with, for example, Field (2000) stating Pearson r is “extremely robust” (p. 87) and Triola (2010) claiming “data must have a bivariate normal distribution” (p. 520). Departures from normality have a tendency to inflate Type I error and reduce power.

Absence of extreme outliers. Pearson r is very sensitive to outliers. A nonparametric test should be used if outliers are detected.

Independence of observations.

Homoscedasticity. The variability in scores for one variable is roughly the same at all values of a second variable.

Linearity. There is a linear relationship between the two variables. This assumption is best evaluated using a scatterplot.

Example Research Question

Is there a relationship between intrinsic motivation and alienation among online university students, $\rho \neq 0$?

Example Null Hypothesis

H_0 : There is no relationship between intrinsic motivation and alienation among online university students, $\rho = 0$.

SPSS Procedures

SPSS > Analyze > Correlate > Bivariate > Pearson

Move two or more variables to the Variables box. The procedure will conduct bivariate correlation tests on each pair of variables. Select *Pearson* as the *Correlation Coefficient* and *Two-tailed* or *One-tailed* for the *Test of Significance*. Select *Flag significant correlations*, if desired.

Click the *Options* button and select *Statistics (Means and standard deviations and Cross-product deviations and covariances)* and *Missing Values*, as needed. Click *Continue*.

Click *OK*.

SPSS Output & Analysis

Use the SPSS > Analyze > Correlate > Bivariate > Pearson procedure and the SPSS dataset *Motivation.sav* located at <http://www.watertreepress.com/stats> if you want to follow along with this analysis. Variables: *intrinsic motivation (intr_mot)*, *alienation (alienation)*.

(Note: only selected SPSS output is presented below.)

Descriptive Statistics (Mean, Standard Deviation, N).

Correlations.

- Provides Pearson r correlation coefficient, 2-tailed significance level, and sample size.
- If requested as an option, SPSS also displays the cross-product of deviations (equal to the sum of the products of mean-corrected variables) and the covariance (an unstandardized measure of the relationship between two variables, equal to the cross-product deviation divided by $N-1$).

Correlations

		Intrinsic Motivation	Alienation
Intrinsic Motivation	Pearson Correlation	1	-.179*
	Sig. (2-tailed)		.020
	N	168	168
Alienation	Pearson Correlation	-.179*	1
	Sig. (2-tailed)	.020	
	N	168	169

*. Correlation is significant at the 0.05 level (2-tailed).

The above SPSS output shows a low but significant inverse relationship between intrinsic motivation and alienation since the significance level (2-tailed) $\leq .05$ (the assumed *a priori* significance level). As one variable increases, the other decreases.

What to Report

As a minimum, the following information should be reported in the results section of any report: null hypothesis that is being evaluated, descriptive statistics (e.g., M , SD , N), statistical test used (i.e., Pearson product-moment correlation test), results of evaluation of test assumptions, and test results. For example, one might report results as follows.

A Pearson product-moment correlation test was conducted to evaluate the null hypothesis that there was no relationship between intrinsic motivation and alienation among online university students ($N = 168$). The assumption of normality of each variable was evaluated using the Kolmogorov-Smirnov test and was found tenable, $p = .20$. The absence of extreme outliers was confirmed using boxplots and a scatterplot provided evidence that the assumption of linearity was tenable.

The results of the test provided evidence that intrinsic motivation ($M = 55.50$, $SD = 15.37$) was inversely related to alienation ($M = 67.14$, $SD = 11.24$), $r(166) = -.18$, $p = .02$. Therefore, there was sufficient evidence to reject the null hypothesis. The coefficient of determination was $r^2 = .03$, indicating that both variables shared only 3 percent of variance in common, which suggests a very slight relationship.

(Note: one might also include a figure of a scatterplot displaying the strength and direction of relationship between the two variables.)

Point-Biserial Correlation (r_{pb})

Purpose

The point-biserial correlation (r_{pb}) is the special case of the Pearson product moment correlation applied to a dichotomous and a continuous variable. The dichotomous variable is coded as 1 when the trait is present and 0 when the trait is not present; e.g., 1 = pass, 0 = fail. The correlation coefficients produced by the SPSS Pearson r correlation procedure is a point-biserial correlation when these types of variables are used. Like Pearson r , it has a value in the range $-1 \leq r_{pb} \leq 1$. The absolute value of the point-biserial correlation coefficient can be interpreted as follows (Hinkle, Wiersma, & Jurs, 1998):

- Little if any relationship < .30
- Low relationship = .30 to < .50
- Moderate relationship = .50 to < .70
- High relationship = .70 to < .90
- Very high relationship = .90 and above

Degrees of freedom. The degrees of freedom for this test is $N - 2$, where N is the number of cases in the analysis.

Alternative Tests

Pearson product moment correlation coefficient – parametric; determines symmetric linear relationship between two interval/ratio variables.

Spearman rank order correlation coefficient – nonparametric; determines monotonic symmetric relationship between two ranked variables.

Kendall's tau-b – nonparametric; determines monotonic symmetric relationship between two ordinal variables, used when the number of rows and number of columns are equal, adjusts for tied pairs, based on concordant-discordant pairs.

Kendall's tau-c – nonparametric; determines monotonic symmetric relationship between two ordinal variables, used when the number of rows and number of columns are not equal, based on concordant-discordant pairs.

Key Assumptions & Requirements

Random selection of samples (probability samples) to allow for generalization of results to a target population.

Variables. One dichotomous and one continuous variable. Absence of restricted range (i.e., data range is not truncated in any variable).

Measurement without error.

Bivariate normality. The continuous variable should have an underlying distribution that is bivariate normal. This indicates that scores on the continuous variable are normally distributed for each value of the dichotomous variable.

Absence of extreme outliers.

Independence of observations.

Homoscedasticity. The variability in scores for one variable is roughly the same at all values of a second variable.

Linearity.

Example Research Question

Is there a relationship between classroom spirit and type course (on-campus, online), $r_{pb} \neq 0$?

Example Null Hypothesis

H_0 : There is no relationship between classroom spirit and type course (on-campus, online), $r_{pb} = 0$.

SPSS Procedures

SPSS > Analyze > Correlate > Bivariate > Pearson

Move variables to the *Variables* box. Select *Pearson* as the *Correlation Coefficient* and *Two-tailed* or *One-tailed* for the *Test of Significance*. Select *Flag significant correlations*, if desired.

Click the *Options* button and select *Statistics* (*Means and standard deviations* and *Cross-product deviations and covariances*) and *Missing Values*, as needed. Click *Continue*.

Click *OK*.

SPSS Output & Analysis

Use the *SPSS > Analyze > Correlate > Bivariate > Pearson* procedure and the *SPSS dataset Community.sav* located at <http://www.watertreepress.com/stats> if you want to follow along with

this analysis. Variables: Type Course (mode), Spirit Posttest (spirit). Options: Cross-Product deviations and covariances.

(Note: only selected SPSS output is presented below.)

Descriptive Statistics (Mean, Standard Deviation, N).

Correlations.

- Provides Pearson r correlation coefficient, 2-tailed significance level, and sample size.
- If requested as an option, SPSS also displays the cross-product of deviations (equal to the sum of the products of mean-corrected variables) and the covariance (an unstandardized measure of the relationship between two variables, equal to the cross-product deviation divided by $N-1$).

Correlations

		Type course	Spirit Posttest
Type course	Pearson Correlation	1	-.495**
	Sig. (2-tailed)		.000
	Sum of Squares and Cross-products	16.748	-132.975
	Covariance	.137	-1.117
	N	123	120
Spirit Posttest	Pearson Correlation	-.495**	1
	Sig. (2-tailed)	.000	
	Sum of Squares and Cross-products	-132.975	4519.925
	Covariance	-1.117	37.983
	N	120	120

** . Correlation is significant at the 0.01 level (2-tailed).

The above SPSS output displays the Pearson product-moment correlation coefficient, $r(118) = -.50$, $p < .001$. These results are equal to r_{pb} since a continuous variable and a dichotomous variable (0 = on-campus and 1 = online) were used in the analysis. The above SPSS output shows a significant inverse relationship between type course and spirit since the significance level (2-tailed) $\leq .05$ (the assumed *a priori* significance level).

What to Report

As a minimum, the following information should be reported in the results section of any report: null hypothesis that is being evaluated, descriptive statistics (e.g., M , SD , N for the continuous variable; observed frequency counts for each category of the dichotomous variable),

statistical test used (i.e., point-biserial correlation test), results of evaluation of test assumptions, and test results. For example, one might report results as follows.

The point-biserial correlation was used to evaluate the null hypothesis that there was no relationship between classroom spirit and type course (traditional on-campus, distance online), $N = 123$. The Shapiro-Wilk test provided evidence that spirit posttest was not normally distributed in the on-campus group, $W(19) = .90$, $p = .045$, and the Kolmogorov-Smirnov test with Lilliefors significance correction showed spirit posttest was normally distributed in the online group, $D(101) = .08$, $p = .17$. The standard coefficients of skewness and kurtosis for the on-campus group were 2.09 and 1.00 respectively, showing a marked positive skew. Skewness and kurtosis for the online group were $-.57$ and $.02$, supporting normality. An examination of boxplots confirmed the absence of extreme outliers in each group and an examination of a scatterplot provided evidence of linearity.

Point-biserial correlation was conducted despite the departure from normality since much of the professional literature maintains that the test is robust to mild to moderate violations of normality but has a tendency to inflate Type I error and reduce power in such situations. The test showed a significant inverse relationship between spirit posttest ($M = 26.48$, $SD = .56$) and type course (on-campus = 20, online = 103), $r_{pb}(118) = -.50$, $p < .001$ when type course was coded as 0 = on-campus and 1 = online. Consequently, there was sufficient evidence to reject the null hypothesis that there was no relationship. Higher posttest spirit was more closely related to on-campus students than online students in the target population. The coefficient of determination, $r_{pb}^2 = .25$, indicated that both variables shared 25 percent of variance in common, which is a moderate effect size.

Internal Consistency Reliability Analysis

Purpose

Internal consistency reliability analysis is a parametric procedure that measures the internal consistency reliability of a scale or subscale. In other words, it is used to evaluate the consistency of results across items within a single instrument that is composed of multiple items. See reliability in Section 1 (Introduction) of this chapter.

Reliability is the extent to which an item, scale, or instrument will yield the same score when administered at different times or locations, or to different populations, assuming the administrations do not differ on relevant variables being measured. Reliability coefficients are forms of correlation coefficients.

The following models of internal consistency reliability are available in SPSS:

- Cronbach's alpha model is based on the average inter-item correlation.
- Split-half model splits the scale into two parts and examines the correlation between the parts.
- Guttman model computes Guttman's lower bounds for true reliability.
- Parallel model assumes that all items have equal variances and equal error variances across replications.
- Strict parallel model makes the assumptions of the parallel model and also assumes equal means across items.